

## **Soft Computing for detecting thermal insulation failures in buildings**

**Javier Sedano<sup>1</sup>, Enrique de la Cal<sup>2</sup>, Leticia Curiel<sup>3</sup>, José  
Ramón Villar<sup>2</sup>, Emilio Corchado<sup>3</sup>**

<sup>1</sup> *Department of Electromechanical Engineering, University of  
Burgos, Burgos, Spain*

<sup>2</sup> *Department of Computer Science, University of Oviedo, Spain*

<sup>3</sup> *Department of Civil Engineering, University of Burgos, Burgos, Spain*

emails: jsedano@ubu.es, delacal@uniovi.es, lcuriel@ubu.es,  
villarjose@uniovi.es, escorchado@ubu.es

### **Abstract**

Improved detection of thermal insulation efficiency in buildings could substantially contribute to reductions in energy consumption and the carbon footprint of domestic heating systems. Thermal insulation standards are now contractual obligations at the construction stage, although they are not standardized in buildings that are already in operation: lighting, occupancy and temperature profiles, air conditioning, and ventilation, all add to the complexity of standardization processes. The identification of thermal insulation failure can help to reduce energy consumption in heating systems. Conventional methods can be greatly improved through the application of hybridized soft-computing techniques to detect thermal insulation failures when a building is in operation. The method proposed in this paper begins by considering local building and heating system regulations as well as the specific features of the climate zone as part of a three-step procedure. Firstly, the dynamic thermal performance of different variables is modelled, which relate to both the building and the climate zone. Secondly, Cooperative Maximum-Likelihood Hebbian Learning is used to extract their relevant features. Finally, neural projections and identification techniques are applied, in order to detect fluctuations in room temperatures and, in consequence, thermal insulation failures. Although a great deal of further research remains to be done in this field, the proposed system is expected to outperform conventional methods described in Spanish building codes that are used to calculate energetic profiles in domestic and residential buildings.

## 1. Introduction

Soft computing represents a collection or set of computational techniques and intelligent system principles in machine learning, computer science and certain engineering disciplines which investigate, simulate, and analyze very complex issues and phenomena in order to solve real-world problems: one such problem is the detection of thermal insulation failure in buildings [1], [2], which requires a multidisciplinary approach [3].

On the one hand, local building regulations need to be analysed in order to profile the premises and the legal specifications of the physical variables. In the case of Spain, building and heating system regulations are adapted to five winter climate zones and five summer climate zones across the entire country. Building materials, insulation widths, materials, and so on, are calculated according to these <winter-zone, summer-zone> parameters. Further market-related factors should also be included: the geometric design and orientation of a building, aesthetic aspects and its internal layout, all of which have a high impact on thermal dynamics. Taken together they define what is known as the topology of the building. The topology and the <winter-zone, summer-zone> parameters give the building its configuration, which is complemented by environmental variables – such as solar radiation, outdoor temperature, wind speed, and so on. Heating and ventilation systems may then be designed and other profiles estimated that relate to occupancy, lighting, small power devices, ventilation, and set-point temperatures.

Nevertheless, predicting the thermal dynamics of a building is a complex task. The dynamic thermal performance of a building has mainly been used to estimate its power requirements. As an example, the difficulties of obtaining a black-box model for a generic building are documented in [4]. Furthermore, [5] cites examples of the errors associated with different kinds of techniques while providing possible solutions. In both cases, the aim of the study is to design an electrical energy distribution device to control electrical heating systems, in order to constrain energy consumption while maintaining comfort. The influence of thermal efficiency is also analysed for a specific building component in [5]. In this case, the dynamic thermal performance of an aluminium roof is analysed and compared with standard roofing materials. The aforementioned works all examine the design of new buildings, which represents the main area of research, whereas far fewer research projects are concerned with the energy efficiency of buildings that are already in operation.

This work represents a step forward in the development of techniques to improve dynamic thermal efficiency in existing buildings through the detection of thermal insulation failure. The proposal is based on a comparison of behavioural fluctuations caused by specific variables with respect to a predefined steady state. Although this may appear simple at first sight, noise due to occupancy or lighting profiles can introduce distortion and can complicate detection. A three-step procedure for testing and validating the model is proposed: firstly, the dynamic thermal behaviour of a specific configuration is calculated using HTB2 software [6]. The outcome of the HTB2 should then be post-processed to obtain a suitable dataset. Subsequently, the dataset is analysed using Cooperative Maximum-Likelihood Hebbian Learning (CMLHL) to extract the dataset structure and key relationships between the variables. A model is then produced, at the modelling stage, to estimate room temperature at a specific configuration. Finally, thermal insulation failure is identified when the temperature error, measured as the difference between room temperature and model output temperature, rises above a certain threshold.

This paper is organised as follows. Section 2 introduces the unsupervised connectionist techniques for analysing the datasets in order to extract their relevant internal structures. Section 3 deals with classical identification techniques used in the system modelling. In Section 4, the problem details and the multi-step procedure are detailed. Finally, the conclusions are set out and comments are made on future lines of work.

## 2. System Analyses Using Unsupervised Connectionist Techniques

### 2.1. Data structure analysis using connectionist techniques

CMLHL [7] is used in this research to analyse the internal structure of datasets that describe the heating process, so as to establish whether it is "sufficiently informative". In the worse case, the experiments have to be performed again in order to gather a sufficiently informative dataset.

CMLHL is a Exploratory Projection Pursuit (EPP) method [8], [9], [10]. In general, EPP provides a linear projection of a dataset, but it projects the data onto a set of basic vectors which help reveal the most interesting data structures; interestingness is usually defined in terms of how far removed the distribution is from the Gaussian distribution [11].

One connectionist implementation is Maximum-Likelihood Hebbian Learning (MLHL) [10], [12]. It identifies interestingness by maximising the probability of the residuals under specific probability density functions that are non-Gaussian. An extended version is the CMLHL [7], [13] model, which is based on MLHL [10], [12] but adds lateral connections [7], [13] that have been derived from the Rectified Gaussian Distribution [11].

Considering an N-dimensional input vector ( $x$ ), and an M-dimensional output vector ( $y$ ), with  $W_{ij}$  being the weight (linking input  $j$  to output  $i$ ), then CMLHL can be expressed [7] as:

1. Feed-forward step:

$$y_i = \sum_{j=1}^N W_{ij} x_j, \forall i \quad (1)$$

2. Lateral activation passing:

$$y_i(t+1) = [y_i(t) + \tau(b - Ay)]^+ \quad (2)$$

3. Feedback step:

$$e_j = x_j - \sum_{i=1}^M W_{ij} y_i, \forall j \quad (3)$$

4. Weight change:

$$\Delta W_{ij} = \eta \cdot y_i \cdot \text{sign}(e_j) |e_j|^{p-1} \quad (4)$$

Where:  $\eta$  is the learning rate,  $\tau$  is the "strength" of the lateral connections,  $b$  the bias parameter,  $p$  a parameter related to the energy function [10], [12] and  $A$  a symmetric matrix used to modify the response to the data [11]. The effect of this matrix is based on the relation between the distances separating the output neurons.

### 2.2. Feature selection and extraction

Feature Selection and extraction [14], [15] entails feature construction, space dimensionality reduction, sparse representations and feature selection among others. They

are all commonly used pre-processing tools in machine learning tasks, which include pattern recognition. Although researchers have grappled with such problems for many years, renewed interest has recently surfaced in feature extraction. A large number of new applications with very large input spaces require space dimensionality reduction to enhance predictor efficiency and effectiveness. Some of these applications include both new and conventional topics such as bioinformatics (DNA microarrays, etc.), remote sensing multi- and hyperspectral imagery, pattern recognition (e.g. handwriting recognition, text processing, web mining), speech processing, artificial vision, industrial applications, and so on.

Our approach to feature selection in this study is based on the dimensionality reduction issue. Initially, we use the projectionist method known as Cooperative Maximum-Likelihood Hebbian Learning (CMLHL) [7], [13], characterized by its capability to impose a sparser representation of each weight vector than other methods such as PCA [16], [17] or Maximum-Likelihood Hebbian Learning (MLHL) [10], [12] and its ability to preserve a degree of global ordering [13], due to the effect of the lateral connections.

### 3. System Modelling Using Classical Identification Algorithms

#### 3.1. The identification criterion

The identification criterion consists in evaluating the best adapted group of candidate models that best describes the dataset gathered for the experiment; i.e., given a certain model  $M(\theta_*)$ , its prediction error may be defined as in Eq. (5). The aim is to obtain a model that complies with the following premise [18]: a good model is one that makes good predictions, and which produces small errors when the observed data is applied, i.e., on any one dataset  $Z^t$  it will calculate the prediction error  $\varepsilon(t, \theta)$ , Eq. (5), in such a way that for any one  $t=N$ , a particular  $\hat{\theta}_N$  (estimated parametrical vector) is selected so that the prediction error  $\varepsilon(t, \hat{\theta}_N)$  in  $t=1,2,3\dots N$ , is minimized.

$$\varepsilon(t, \theta_*) = y(t) - \hat{y}(t | \theta_*) \quad (5)$$

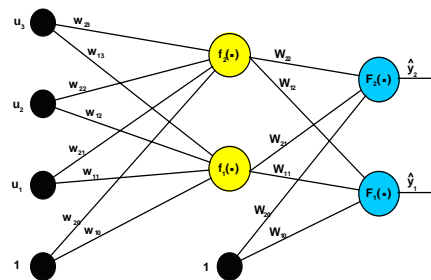
Black-box methods [18] are used either for linear systems or for systems that are linear in the working area; this methodology has the advantage of only requiring very few explicit assumptions on the pattern to be identified, although that in turn makes it difficult to quantify the model that is obtained. The discrete linear models may be represented through the union of a deterministic model and a stochastic model, Eq. (6). In Eq. (6),  $u(t)$  is the input,  $y(t)$  is the output,  $G(q^{-1})$  is the transfer function from  $u(t)$  to  $y(t)$ ,  $H(q^{-1})$  is the transfer function from  $e(t)$  to  $y(t)$  and  $q^{-1}, q$  are backward and forward shift operators. The term  $e(t)$  (white noise signal) includes the modelling errors and is associated with a series of random variables, of mean null value and variance  $\lambda$ .

$$y(t) = G(q^{-1})u(t) + H(q^{-1})e(t) \quad (6)$$

The structure of a black-box model depends on how the noise influences the model [18], that is, the term  $H(q^{-1})$ . Thus, if this term is 1, then the FIR (Finite Impulse Response) and OE (Output Error) models are applicable; whereas if it is different from zero a great range of models are applicable; the most common being: ARX (Autoregressive with external input), ARMAX (Autoregressive Moving Average with external input), BJ (Box Jenkins) and ARMA (Autoregressive Moving Average).

### 3.2. The ANN in the identification process

The use of artificial neural networks (ANN) in the process of identification requires the selection of several parameters: the number of layers, the number of neurons per layer and the activation functions. The methods to set up the parameters have been documented in depth in the literature. It was found that ANN with two layers using sigmoidal or hyperbolic functions in the hidden layer are universal approximators or predictors [19], [20]. A Multilayer Perceptron (MLP) network with two layers is shown in Figure 1.



**Fig. 1.** A MLP network with two layers, with two nodes per layer, and three inputs.  $W$  is the weight matrix between the hidden and output layer, while  $w$  is the weight matrix between the inputs and the hidden layer. The network has two bias nodes with value 1.

The number of neurons per layer is also a relevant design parameter that should be analyzed in order to avoid over fitting [21], [22]; which state that the number of neurons and the size of the weight matrix depends on the ANN training algorithm. Each algorithm will introduce some restrictions in the weight matrix. The most widely used training algorithms in system identification are the Levenberg-Marquardt method [23], recursive Gauss-Newton method [18], the batch and the recursive versions of the back-propagation algorithm [24].

### 3.3. The process of identification

When using ANN, the purpose of an identification process is to determine the weight matrix based on the observations  $Z^t$ , so that the relationships between nodes in the network are given. The weight matrix is usually referred as  $w$ ,  $W$  or  $\theta$ .

The supervised learning algorithm is then applied to find the estimator  $\theta$ , so as to obtain the identification criterion. In this case, the minimization of the mean square error criterion as defined in Eq. (7) and (8) is used. The iterative minimization scheme is defined in Eq. (9), where  $f(t)$  is the search direction and  $\mu(t)$  step size.

$$V_N(\theta, z^t) = \frac{1}{2N} \sum_{t=1}^N [y(t) - \hat{y}(t|\theta)]^T [y(t) - \hat{y}(t|\theta)] \quad (7)$$

$$\hat{\theta} = \arg \min_{\theta} V_N(\theta, Z^t) \quad (8)$$

$$\theta(t+1) = \theta(t) + \mu(t)f(t) \quad (9)$$

Several well-known model structures are used when merging system identification with ANN. If the ARX model is used as the regression vector  $\theta$ , the model structure is called NNARX, as can be seen in Eq. (10). NNARX stands for neural network ARX. Likewise, NNFIR, Eq. (11), NNARMAX, Eq. (12) and NNOE structures, Eq. (13), are also used in depth. In the same way, it is possible to use an estimator for the one-step prediction ahead of the output  $\hat{y}(t, \theta)$ , i.e., the NNARX, Eq. (14), the NNFIR, Eq. (14), the NNARMAX, Eq. (15) and the NNOE, Eq. (15). The polynomial degree values  $n_a$ ,  $n_b$ ,  $n_c$ ,  $n_d$ ,  $n_f$  and  $n_k$  are given as parameters.

$$\varphi(t) = [y(t-1) \dots y(t-n_a) u(t-n_k) \dots u(t-n_b-n_k+1)]^T \quad (10)$$

$$\varphi(t) = [u(t-n_k) \dots u(t-n_b-n_k+1)]^T \quad (11)$$

$$\varphi(t) = [y(t-1) \dots y(t-n_a) u(t-n_k) \dots u(t-n_b-n_k+1) e(t-1) \dots e(t-n_c)]^T \quad (12)$$

$$\varphi(t) = [\hat{y}(t-1|\theta) \dots \hat{y}(t-n_a|\theta) u(t-n_k) \dots u(t-n_b-n_k+1)]^T \quad (13)$$

$$\hat{y}(t|\theta) = \hat{y}(t|t-1, \theta) = g(\varphi(t), \theta) \quad (14)$$

$$\hat{y}(t|\theta) = g(\varphi(t), \theta) \quad (15)$$

### 3.4. The system identification methodology

The best model for estimating the thermodynamic conditions must be chosen. The identification procedure used to generate the final model entails setting the identification techniques [18], [25], [26], [27], [28], selecting the model structure, estimating the most appropriate polynomial degree [29], [30], the identification criterion, and the optimization techniques.

The identification procedure also includes a validation stage, which ensures that the selected model meets the necessary conditions for estimation and prediction. In order to validate the model, three tests were performed: residual analysis  $\varepsilon(t, \hat{\theta}(t))$ , by means of a correlation test between inputs, residuals and their combinations; final prediction error (FPE) estimate as explained by Akaike [31] and finally a graphical comparison between desired outputs and the outcome of the models through simulation one (or k) steps before.

## **4. The multi-step method for detecting thermal insulation failures in buildings**

### **4.1. Thermal dynamics data gathering by means of simulation**

A three-step method is proposed to detect thermal insulation failures in buildings. Firstly, a model of a building's dynamic thermal performance is determined, under normal operation. This data can be gathered in two ways: either by distributing sensors around the building or by simulation. Once the thermodynamics data have been gathered, then a model for normal operation may be obtained. Finally, thermal insulation failures can be detected using this model whenever significant fluctuations in room temperature are identified.

This sub-section deals with the method of detecting thermal insulation failures, and the next sub-section sets out the procedure used to gather thermodynamic data by simulation. It is then shown how CMLHL is used to extract features from the thermodynamic data using relevant information from the process. Finally, the system identification techniques are applied to obtain the most appropriate model used to detect thermal insulation failures. Two methods are used to collect thermodynamic data on a building: either through a network of sensors placed in spaces of scale-model or actual building to measure the desired variables; or by using specific thermodynamic simulation software [6]. This second choice is more suitable than physical modelling due to its flexibility and scalable solutions that saves resources.

The following elements have to be defined to simulate the dynamic thermal behavior of a building:

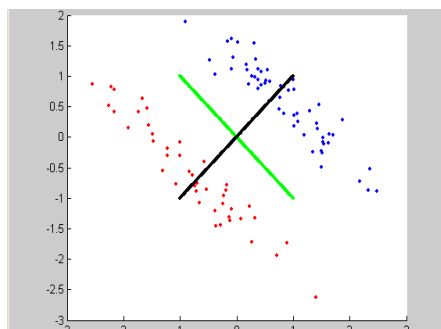
- Building topology: internal layout and orientation.
- Climate zone: the Spanish regulation sets five winter/summer zones, from E1 (more severe climate zones) to A3 (gentler climate zone).
- Meteorological data for the climate zone and the simulated time period: such as solar radiation, outdoor temperature, wind speed, etc. Meteorological data is generally available from weather stations in the climate zone.
- Building materials that comply with local regulations for the chosen climate zone.
- Heating profile: the power supply in each space as per local regulations and the user-defined set point temperature profiles.
- Lighting profile: it is necessary to define the operating profile of the lighting systems in each space as their heat influences the thermodynamic behaviour of the building.
- Small power devices profile: in the same way as for lighting, small power sources (typically electric appliances) produce heat when in use, affecting the dynamic thermal behavior of the building.
- Occupancy profile: occupancy heat gains for each space must also be factored in, as well as the metabolic rates (sensible heat gain per adult in W) that correspond to the activities in the space. Occupancy timetables for each space must therefore be defined.
- Ventilation profile: ventilation and infiltration are important factors in the energetic behaviour of buildings. There are several ventilation specifications for a building; from specifying air change rates in spaces, to specifying space to space air flows and individual window openings.

Having defined and/or gathered these data sets, the chosen simulation tool is applied to obtain the output data, which is needed for the next step: regulation of the air temperature in each space and calculation of the output power for each heater to meet the heating profiles.

## 4.2. Extraction of the relevant internal structures

As may be seen from figure 2, CMLHL is a powerful technique for identifying internal dataset structures. It is applied to a dataset, in order to select the features that best describe the relationships between the heating conditions, and in order to establish whether the dataset is sufficiently informative. The results of applying the method are shown in Figure 2.

Having analysed the overall global results, it is clear from Figure 2 that the examples can be classified in two different clusters, and the dataset may be said to have an interesting internal structure. When the dataset is considered sufficiently informative, the third step in the process begins. This step performs an accurate and efficient optimization of the heating system model to detect thermal insulation failures in the building, through the application of several conventional modelling systems.



**Fig. 2.** The CMLHL projection shows the internal structure of a dataset.

## 4.3. System identification to model normal building operation

Once the relevant variables and their transformation have been extracted from the thermal dynamics data, then a model to fit the normal building operation should be obtained in order to identify the bias in the room temperature, which, at the end, is used for failure detection. The different model learning methods used were implemented in Matlab© making use of several toolboxes: the System Identification Toolbox, the Control System Toolbox, the Neural Network Toolbox and the Neural network Based System Identification Toolbox [32]. The experiment followed the identification procedure detailed in Section 3.4: the model structures were analyzed in order to obtain the models that best suited the dataset. The Akaike Information Criterion (AIC) was used to obtain the best degree of the model and its delay for each model structure. A total of 70 techniques were carried out to obtain the models, including:

- The frequency response analysis based on the spectrum analysis and the Fourier Fast Transform (FFT) were used to determine the data dynamics,
- The finite impulse response method (FIR) correlation analysis was used to determine the steady state conditions,



- Black-box techniques: up to 31 different combinations of model structure and optimization techniques were considered - such as the least-squares method, QR factorization of ARX models, or the recursive normalized gradient algorithm of RARMAX models [18], [26],
- The nonlinear model structures synthesis: up to 34 different combinations of model structures and optimization techniques were considered-such as the Levenberg-Marquardt method, the batch version of the back-propagation algorithm or recursive Gauss-Newton method for NNARX, NNFIR, NNARMAX and NNOE models [18], [23], [24],
- Three different residual analyses based on cross correlation were carried out: residual analysis between the residual  $\hat{R}_e^N(\tau)$ , between the residual and the input  $\hat{R}_{eu}^N(\tau)$  and the non-linear residual correlation  $\hat{R}_{e^2,u^2}^N(\tau)$ .

Several different indexes have been used to validate the obtained models. The indexes are well-known and widely used measures in system identification [18], [25], [26]:

- The percentage representation of the estimated model: calculated as the normalized mean error for the one-step prediction (FIT1), for the ten-step prediction (FIT10) and with the  $\infty$ -step prediction (FIT). The FIT is known as simulation in classical system identification.
- The graphical representation of the FIT1 –  $\hat{y}_1(t|m)$ –, the FIT10 –  $\hat{y}_{10}(t|m)$ – or the FIT –  $\hat{y}_\infty(t|m)$ –.
- The loss function or error function (V): the numeric value of the mean square error that is computed using the estimation dataset.
- The generalization error value (NSSE): the numeric value of the mean square error that is computed with the validation dataset.
- The FPE calculated as the average generalization error value computed with the estimation dataset.

## 5. Conclusions and future work

Effective thermal insulation is an essential component of energy efficient heating systems in buildings. The more effective the insulation in the buildings, the lower the energy losses due to insulation failures. Thus, the possibility of improving the detection of thermal insulation failures represents a fresh challenge for building energy management. The new methodology proposed in this study for detecting thermal insulation failures entails either a network of sensors in the building or specialized simulation software in cases where no such network is available. Finally, different techniques are applied to obtain a suitable model which will be responsible for detecting the failures as a fluctuation of predicted room temperature. Future work will include testing the proposal in different climate zones and in different building types (residential, domestic) to develop generic methods.

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